Changes in Frequencies of a Laminated Plate Caused by Embedded Piezoelectric Layers

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Introduction

THE three-dimensional linear theory of elasticity is used to delineate the effect of the thickness, stiffness, and mass density of piezoelectric (PZT) layers on the first five natural frequencies of a simply supported graphite/epoxy laminated plate with PZT layers embedded in it.

Platelike smart structures are usually made of PZT patches either embedded in or bonded to the bounding surfaces of a plate. In this Note, an attempt is made to ascertain changes in the natural frequencies of the underlying plate due to the PZT patches.

Problem Formulation

Consider a simply supported graphite/epoxy laminated plate with PZT layers embedded in it. The plate is assumed to be made of a homogeneous, orthotropic, linear elastic material and the PZT of a homogeneous, transversely isotropic, linear piezoelectric material poled in the thickness direction. Their constitutive relations are

$$\boldsymbol{\tau} = \boldsymbol{C}\boldsymbol{s} - \boldsymbol{e}^T\boldsymbol{E} \tag{1}$$

$$\boldsymbol{D} = \boldsymbol{e}\boldsymbol{s} + \boldsymbol{\epsilon}\boldsymbol{E} \tag{2}$$

where $\boldsymbol{\tau}$ is the stress tensor, $\boldsymbol{s}$ the infinitesimal strain tensor, $\boldsymbol{E}$ the electric field vector, $\boldsymbol{D}$ the electric displacement vector, $\boldsymbol{C} = \boldsymbol{C}^T$ the material elasticity tensor, $\boldsymbol{e}$ the piezoelectric constants, and ϵ the dielectric permittivity. Equation (1) with $\boldsymbol{e} = \boldsymbol{0}$ gives the stress–strain relation for each graphite/epoxy lamina.

Batra et al.[1,2] and Batra and Liang[3] have used the three-dimensional linear elasticity theory to study vibrations of a rectangular, simply supported composite plate with PZT layers either embedded in it or bonded to its upper and/or lower surfaces. The same approach is followed here. The PZT layers are assumed to be perfectly bonded to the substrate and are modeled as thin films. The structure is excited by applying a sinusoidal voltage to a PZT actuator.

Results

Results presented herein are for a $30\sqrt{2} \times 30 \times 0.404$ cm, $T300/976$ graphite/epoxy laminated plate. The mass density of the graphite/epoxy plate equals 1600 kg/m^3, and values in MPa assigned to its nonzero elasticities with coordinate axes coincident with the principal axes of the lamina are $C_{11} = 152{,}350$, $C_{12} = C_{13} = 391.8$, $C_{22} = C_{33} = 10{,}000$, $C_{23} = 306.75$, $C_{44} = 250.05$, and $C_{55} = C_{66} = 709.95$. The mass density of the PZT equals 7500 kg/m^3; its nonvanishing dielectric constants in coulomb per square meter, with x_3 (thickness) being the poling direction, are $e_{31} = e_{32} = -2.1$, $e_{33} = 9.5$, and $e_{24} = e_{15} = 9.2$; and its nonzero elasticities in gigapascal are $C_{11} = 148$, $C_{12} = 76.2$, $C_{13} = 74.2$, $C_{33} = 131$, and $C_{66} = 35.9$. In Eqs. (1) and (2), $\boldsymbol{\tau}$ and $\boldsymbol{s}$ have been regarded as six-dimensional vectors so that $\boldsymbol{C}$ is a 6×6 symmetric matrix. The plate is made of 10 graphite/epoxy lamina 0.4 mm thick with fiber orientations $[0/90/0/90/0\ \mathrm{deg}]_s$ and two PZT layers, each 0.02 mm thick, embedded in it. The PZT layers are located between the second and the third substrate layers from the bottom and between the second and the third substrate layers from the top.

The nondimensional natural frequencies Ω_{mn} of the plate are related to its dimensional natural frequencies ω_{mn} by $\Omega_{mn} = \omega_{mn}/(\pi^2/\ell^2)(D_{11}/2\rho h)^{1/2}$, where $2h$ is the thickness of the plate, ℓ its length, and D_{11} the flexural rigidity. For the graphite/epoxy plate, the five lowest nondimensional natural frequencies Ω_{11}, Ω_{21}, Ω_{12}, Ω_{22}, and Ω_{31} according to the thin plate theory[4] equal 2.002, 4.585, 6.408, 8.007, and 9.454, respectively.

Figures 1 and 2 depict, respectively, the dependence of the natural frequency of the composite plate with the PZT layers divided by the corresponding frequency of the plate only on the variation in the thickness and the mass density of the PZT layers. In each case, the abscissa is nondimensionalized by dividing the value of a variable for the PZT layer by that for the plate. Because different values are used to nondimensionalize each frequency, the curve corresponding to the first frequency is not below those for the other frequencies. Each one of the five frequencies decreases monotonically, though not at the same rate, with an increase in the thickness of the PZT layer. For a given thickness of the PZT layer, the relative change in the fifth lowest frequency is the highest, that in the lowest fourth frequency is between those for the first and third frequencies, and that in the second frequency is between those for the third and fifth lowest frequencies. For results plotted in Fig. 2, the thickness of the PZT layer was set equal to that of a lamina. Note that, in the experimental setup of Moetakef et al.,[5] the thickness of the PZT patch equaled nearly one-half of that of the beam. The relative change in the lowest five frequencies decreases monotonically

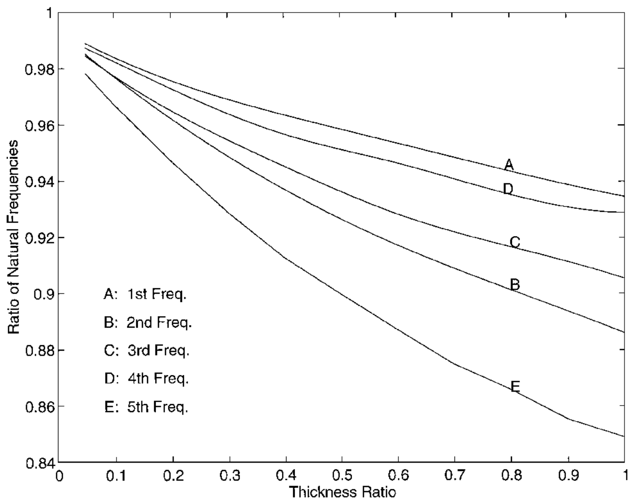

Fig. 1 Frequency of the plate with two PZT layers/frequency of the plate without the PZT layers vs thickness of the PZT layer/thickness of the lamina.

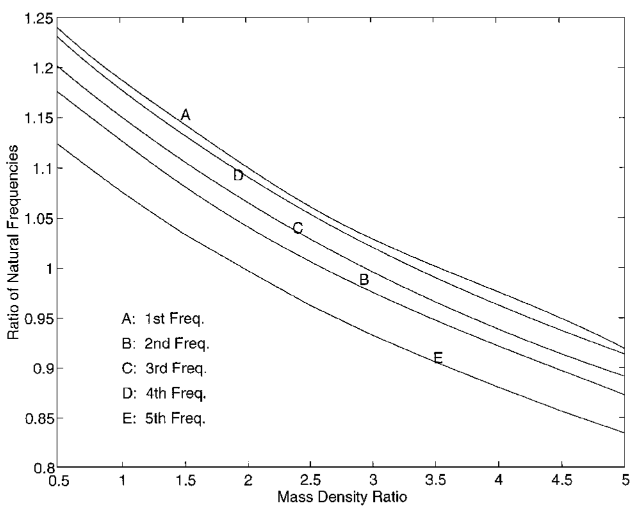

Fig. 2 Frequency of the plate with two PZT layers/frequency of the plate without the PZT layers vs mass density of the PZT layer/mass density of the laminate.

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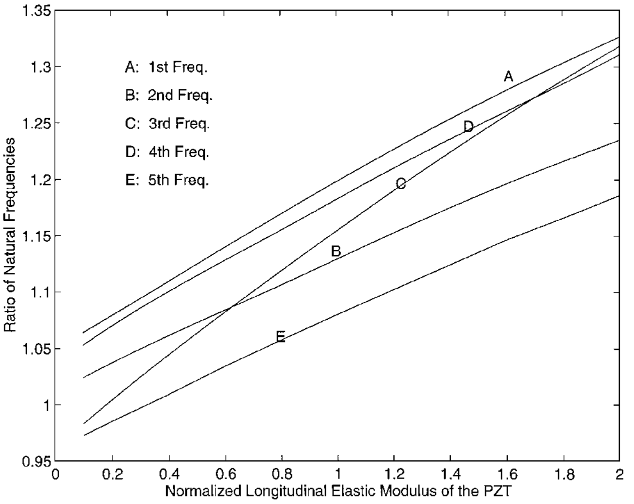

Fig. 3 Frequency of the plate with two PZT layers/frequency of the plate without the PZT layers vs C_{11} for the PZT layer/C_{11} for the 0-deg lamina.

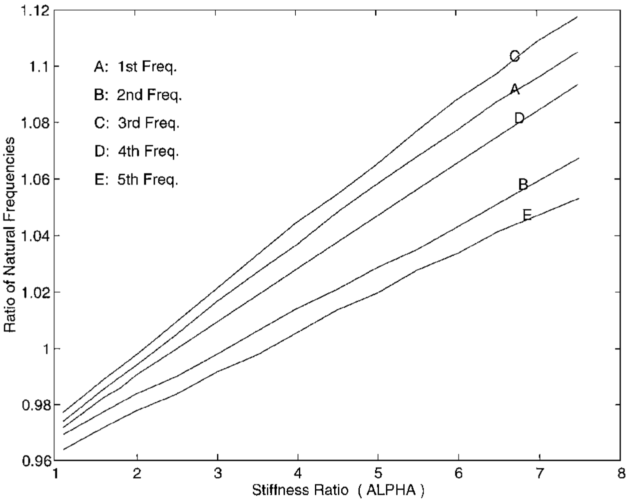

Fig. 4 Frequency of the plate with two PZT layers/frequency of the plate without the PZT layers vs α.

with an increase in the mass density of the PZT layer. The five curves are nearly parallel to each other, signifying that the rate of change of frequency with respect to the mass density of the PZT layer is the same for all five frequencies. As for the variation in the thickness of the PZT layer, the curve for the fourth frequency lies between those for the first and the third ones and that for the third frequency lies between those for the third and fifth ones.

Because the substrate layer has been modeled as orthotropic and the PZT layer as transversely isotropic, it is not clear how to vary the stiffness of the PZT layer relative to that of the graphite/epoxy substrate. In an attempt to decipher the effect of the material stiffness only, the mass density and the thickness of the PZT layer are first set equal to that of the graphite/epoxy lamina, and only C_{11} for the PZT layer is varied. Smart structure's response is also affected by values of other components of $\boldsymbol{C}$ for the PZT. As shown in Fig. 3, all five lowest natural frequencies of the composite structure increase monotonically with an increase in the value of C_{11} for the PZT. In the second study of the effect of the PZT stiffness on the natural frequencies of the plate, the thickness of the PZT layer is taken to be equal to $\frac{1}{10}$th of the thickness of the substrate layer, which is more likely to occur in a physical situation, mass density of the PZT layer equal to 7500 kg/m^3, $\boldsymbol{C}^{\mathrm{PZT}} = \alpha \boldsymbol{C}^{\mathrm{substrate}}$, and α is varied. Of course, it may be difficult to manufacture a PZT with such mechanical properties. Figure 4 illustrates the effect of such a change in the elastic moduli of the PZT on the five lowest natural frequencies of the composite plate. A sevenfold increase in the stiffness of the PZT increases the first five natural frequencies by at most 10%.

Conclusions

For the simply supported composite plate studied herein, it is found that an increase in the thickness of the PZT layer or its mass density monotonically decreases each one of the five lowest frequencies and an increase in the stiffness of the PZT layer relative to that of the lamina increases these frequencies. However, the relative change in these frequencies is not necessarily the same.

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References

[1] Batra, R. C., Liang, X. Q., and Yang, J. S., "The Vibration of a Simply-Supported Rectangular Elastic Plate Due to Piezoelectric Actuators," *International Journal of Solids and Structures*, Vol. 33, No. 11, 1996, pp. 1597–1618.

[2] Batra, R. C., Liang, X. Q., and Yang, J. S., "Shape-Control of Vibrating Simply Supported Plates," *AIAA Journal*, Vol. 34, No. 1, 1996, pp. 116–122.

[3] Batra, R. C., and Liang, X. Q., "The Vibration of a Rectangular Laminated Elastic Plate with Embedded Piezoelectric Sensors and Actuators," *Computers and Structures*, Vol. 63, No. 2, 1997, pp. 203–216.

[4] Jones, R. M., *Mechanics of Composite Materials*, Scripta, Washington, DC, 1975, Chap. 4.

[5] Moetakef, M. A., Joshi, S. P., and Lawrence, K. L., "Elastic Wave Generation by Piezoceramic Patches," *AIAA Journal*, Vol. 34, No. 10, 1996, pp. 2110–2117.

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Bifurcation Buckling Analysis of Delaminated Composites Using Global–Local Approach

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I. Introduction

THE buckling and postbuckling analyses of laminated composites with multiple delaminations are important because a low-speed impact generates multiple delaminations through the laminates. However, an analytical solution approach is only possible for simple delamination zone geometry and boundary conditions because the buckling and postbuckling problems are complicated.[1] Therefore, the finite element method is preferable to analyze the buckling behavior of multiply delaminated composites for arbitrary layup configurations, boundary conditions, loading conditions, and delaminated zone shapes.

Lee et al. analyzed the bifurcation buckling[2] and postbuckling behavior[3] of a laminated composite beam with layerwise finite elements proposed by Reddy.[4] However, for multiple-delaminated composites with layer-dependent finite elements, a large number of degrees of freedom (DOF) is required through the thickness because of the geometric complexity of the delaminated zones. Thus, much computer memory and computing time are required. Furthermore, the stability analysis of laminated structures is a nonlinear problem

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